Problem Set III: Due at last class

1.) a.) Consider a chunk of collisionless, self-gravitating matter in one dimension. Here, take a "chunk" to be:

$$f = \begin{cases} f_0, & u_0 - \Delta v < v < u_0 + \Delta v \\ 0, & \text{elsewhere} \end{cases}.$$

Here, f_0 is constant. Take $u_0, \Delta v$ fixed. Using the Vlasov-Poisson system, calculate the marginal stability criterion for Jeans instability. Compare your result to the case discussed in class for a self-gravitating gas.

b.) Now consider a plasma, with

$$f = \begin{cases} f_{\max} + f_0, & u_0 - \Delta v < v < u_0 + \Delta v \\ f_{\max}, & \text{elsewhere} \end{cases}.$$

Consider $f_0 > 0$ and $f_0 < 0$. $f_{\max}$ is the usual Maxwellian. Of course $f_{\max} + f_0 > 0$, for all v. What is the marginality condition now? Relate your result to the bunching condition discussed in class for the beam-plasma interaction. Hint: Consider the sign of the dielectric function.

c.) For collisionless, self-gravitating matter with an initially Jeans unstable distribution, discuss how the instability might saturate. Hint: Consider simple quasi-linear analysis.

2.) a.) Using the Fokker-Planck equation and the Rosenbluth potentials, calculate the slowing down time for a beam with velocity $V_0 < v_{Ti} < v_{Te}$ impinging on a plasma. Compare your result with the two cases discussed in class.

b.) Compare the classical slowing down time for a beam with $V_0 > v_{Te} > v_{Ti}$ with the growth rate of a two-stream instability if $V_0/L < \omega_p$. Assume $n_b < n_0$. What constraint does stability impose on design of the beam-plasma system?

3.) a.) Derive the *resonant* quasilinear diffusion equation using a Fokker-Planck calculation.

b.) Explain carefully why the non-resonant diffusion effect *cannot* be derived using a Fokker-Planck approach.

4.) a.) Calculate the electron and ion heating predicted by quasi-linear theory for a spectrum of unstable ion-acoustic waves. Which is larger?

b.) How does the electron heating rate compare to the seat-of-the-pants estimate ηJ^2, using the *anomalous* resistivity (from quasi-lineary theory) for η.